key

Math 19 Problem Set 10: p 245 Ex. 1ac, 2ac

Verify $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$, $r=0$

1.(a) $u(t,x) = \frac{2}{\sqrt{t}} e^{-x^2/4\mu t} + \frac{1}{\sqrt{t}} e^{-(x-1)^2/4\mu t}$

$$\frac{\partial u}{\partial t} = -t^{-3/2} e^{-x^2/4\mu t} + \frac{2}{\sqrt{t}}\left(\frac{x^2}{4\mu t^2}\right) e^{-x^2/4\mu t} - \frac{1}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} + \frac{1}{\sqrt{t}}\left(\frac{(x-1)^2}{4\mu t^2}\right) e^{-(x-1)^2/4\mu t}$$

$$= -t^{-3/2} e^{-x^2/4\mu t} + \frac{x^2}{2\mu t^{5/2}} e^{-x^2/4\mu t} - \frac{1}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} + \frac{(x-1)^2}{4\mu t^{5/2}} e^{-(x-1)^2/4\mu t}$$

$$\frac{\partial u}{\partial x} = \frac{2}{\sqrt{t}}\left(-\frac{2x}{4\mu t}\right) e^{-x^2/4\mu t} + \frac{1}{\sqrt{t}}\left(\frac{-2(x-1)}{4\mu t}\right) e^{-(x-1)^2/4\mu t}$$

$$= -\frac{x}{\mu t^{3/2}} e^{-x^2/4\mu t} - \frac{x-1}{2\mu t^{3/2}} e^{-(x-1)^2/4\mu t}$$

$$\frac{\partial^2 u}{\partial x^2} = -\frac{x}{\mu t^{3/2}}\left(-\frac{2x}{4\mu t}\right) e^{-x^2/4\mu t} - \frac{1}{\mu t^{3/2}} e^{-x^2/4\mu t} - \frac{x-1}{2\mu t^{3/2}}\left(\frac{-2(x-1)}{4\mu t}\right) e^{-(x-1)^2/4\mu t} - \frac{1}{2\mu t^{3/2}} e^{-(x-1)^2/4\mu t}$$

$$= \frac{x^2}{2\mu^2 t^{5/2}} e^{-x^2/4\mu t} - \frac{1}{\mu t^{3/2}} e^{-x^2/4\mu t} + \frac{(x-1)^2}{4\mu^2 t^{5/2}} e^{-(x-1)^2/4\mu t} - \frac{1}{2\mu t^{3/2}} e^{-(x-1)^2/4\mu t}$$

$$\mu \frac{\partial^2 u}{\partial x^2} = \frac{x^2}{2\mu t^{5/2}} e^{-x^2/4\mu t} - t^{-3/2} e^{-x^2/4\mu t} + \frac{(x-1)^2}{4\mu t^{5/2}} e^{-(x-1)^2/4\mu t} - \frac{1}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} = \frac{\partial u}{\partial x}$$

(c) $u(t,x) = \frac{2}{\sqrt{t}} e^{-x^2/4\mu t} + \frac{3}{\sqrt{t}} e^{-(x-1)^2/4\mu t}$

$$\frac{\partial u}{\partial t} = -t^{-3/2} e^{-x^2/4\mu t} + \frac{x^2}{2\mu t^{5/2}} e^{-x^2/4\mu t} - \frac{3}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} + \frac{3(x-1)^2}{4\mu t^{5/2}} e^{-(x-1)^2/4\mu t}$$

$$\frac{\partial u}{\partial x} = -\frac{x}{\mu t^{3/2}} e^{-x^2/4\mu t} - \frac{3(x-1)}{2\mu t^{3/2}} e^{-(x-1)^2/4\mu t}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{x^2}{2\mu^2 t^{5/2}} e^{-x^2/4\mu t} - \frac{1}{\mu t^{3/2}} e^{-x^2/4\mu t} + \frac{3(x-1)^2}{4\mu^2 t^{5/2}} e^{-(x-1)^2/4\mu t} - \frac{3}{2\mu t^{3/2}} e^{-(x-1)^2/4\mu t}$$

$$\mu \frac{\partial^2 u}{\partial x^2} = \frac{x^2}{2\mu t^{5/2}} e^{-x^2/4\mu t} - t^{-3/2} e^{-x^2/4\mu t} + \frac{3(x-1)^2}{4\mu t^{5/2}} e^{-(x-1)^2/4\mu t} - \frac{3}{2} t^{-3/2} e^{-(x-1)^2/4\mu t} = \frac{\partial u}{\partial x}$$

2.(a) $u(t,x) = 2e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} + 3e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}}$

$$u_t = 2\lambda e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} + 3\lambda t e^{-x(\lambda/\mu)^{1/2}} = \lambda u$$

$$u_x = 2\left(\frac{\lambda}{\mu}\right)^{1/2} e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} - 3\left(\frac{\lambda}{\mu}\right)^{1/2} e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}}$$

$$u_{xx} = \frac{2\lambda}{\mu} e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} - \frac{3\lambda}{\mu} e^{\lambda t} e^{-x(\lambda/\mu)^{1/2}} = \frac{\lambda}{\mu} u$$

$$\mu u_{xx} = \lambda u = u_t$$

(c) $u(t,x) = 2e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} + 1 - 5x$

$$u_t = 2\lambda e^{\lambda t} e^{x(\lambda/\mu)^{1/2}}$$

$$u_x = 2\left(\frac{\lambda}{\mu}\right)^{1/2} e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} - 5$$

$$u_{xx} = 2\left(\frac{\lambda}{\mu}\right) e^{\lambda t} e^{x(\lambda/\mu)^{1/2}}$$

$$\mu u_{xx} = 2\lambda e^{\lambda t} e^{x(\lambda/\mu)^{1/2}} = u_t$$