Math 19. Problem Set # 1 Solutions
Ex. 1, 2, 4 (a, b, c, d), 5, page 43-44

October 2, 2004

Ex. 1

a)

$\frac{dy}{dt} = 5y$
$\int \frac{dy}{y} = \int 5dt$
$\ln y = 5t + C$
$y = e^{5t+C} = C \cdot e^{5t}$

b)

$\frac{dy}{dt} = -3y$
$\int \frac{dy}{y} = \int (-3)dt$
$\ln y = -3t + C$
$y = e^{-3t+C} = C \cdot e^{-3t}$

c)

$\frac{dy}{dt} = 12y$
$\int \frac{dy}{y} = \int 12dt$
$\ln y = 12t + C$
$y = e^{12t+C} = C \cdot e^{12t}$

d)

$\frac{dy}{dt} = -1.5y$
$\int \frac{dy}{y} = \int (-1.5)dt$
$\ln y = -1.5t + C$
$y = e^{-1.5t+C} = C \cdot e^{-1.5t}$

Where C is a constant.

You can also just use the formula for the exponential growth equation, going directly from $\frac{dp}{dt} = ap$ to $p(t) = p(0) \cdot e^{at}$.

Thus, you can write:

a)

$\frac{dy}{dt} = 5y$
$y(t) = y(0) \cdot e^{5t}$

b)

$\frac{dy}{dt} = -3y$
$y(t) = y(0) \cdot e^{-3t}$

c)

$\frac{dy}{dt} = 12y$
$y(t) = y(0) \cdot e^{12t}$

d)

$\frac{dy}{dt} = -1.5y$
$y(t) = y(0) \cdot e^{-1.5t}$

Ex. 2

a)

$y(0) = 1$

1a) $y(t) = y(0) \cdot e^{5t}$
$y(t) = e^{5t}$

1b) $y(t) = y(0) \cdot e^{-3t}$
$y(t) = e^{-3t}$

1c) $y(t) = y(0) \cdot e^{12t}$
$y(t) = e^{12t}$

1d) $y(t) = y(0) \cdot e^{-1.5t}$
$y(t) = e^{-1.5t}$

b)

$y(1) = 1$

1a) $y(t) = y(0) \cdot e^{5t}$
$1 = y(0) \cdot e^{5 \cdot 1}$
$y(0) = e^{-5}$
$y(t) = e^{-5+5t}$

1b) $y(t) = y(0) \cdot e^{-3t}$

$1 = y(0) \cdot e^{-3 \cdot 1}$

$y(0) = e^{3}$

$y(t) = e^{3-3t}$

1c) $y(t) = y(0) \cdot e^{12t}$

$1 = y(0) \cdot e^{12 \cdot 1}$

$y(0) = e^{-12}$

$y(t) = e^{-12+12t}$

1d) $y(t) = y(0) \cdot e^{-1.5t}$

$1 = y(0) \cdot e^{-1.5 \cdot 1}$

$y(0) = e^{1.5}$

$y(t) = e^{1.5-1.5t}$

c)

$y(-1) = 1$

1a) $y(t) = y(0) \cdot e^{5t}$

$1 = y(0) \cdot e^{-5 \cdot 1}$

$y(0) = e^{5}$

$y(t) = e^{5+5t}$

1b) $y(t) = y(0) \cdot e^{-3t}$

$1 = y(0) \cdot e^{3 \cdot 1}$

$y(0) = e^{-3}$

$y(t) = e^{-3-3t}$

1c) $y(t) = y(0) \cdot e^{12t}$

$1 = y(0) \cdot e^{-12 \cdot 1}$

$y(0) = e^{12}$

$y(t) = e^{12+12t}$

1d) $y(t) = y(0) \cdot e^{-1.5t}$

$1 = y(0) \cdot e^{1.5 \cdot 1}$

$y(0) = e^{-1.5}$

$y(t) = e^{-1.5-1.5t}$

d)

$y(-1) = -1$

1a) $y(t) = y(0) \cdot e^{5t}$
$-1 = y(0) \cdot e^{-5 \cdot 1}$
$y(0) = -e^{5}$
$y(t) = -e^{5+5t}$

1b) $y(t) = y(0) \cdot e^{-3t}$
$-1 = y(0) \cdot e^{3 \cdot 1}$
$y(0) = -e^{-3}$
$y(t) = -e^{-3-3t}$

1c) $y(t) = y(0) \cdot e^{12t}$
$-1 = y(0) \cdot e^{-12 \cdot 1}$
$y(0) = -e^{12}$
$y(t) = -e^{12+12t}$

1d) $y(t) = y(0) \cdot e^{-1.5t}$
$-1 = y(0) \cdot e^{1.5 \cdot 1}$
$y(0) = -e^{-1.5}$
$y(t) = -e^{-1.5-1.5t}$

Ex. 4 The first order Taylor's approximation:

$g(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$
Since $x_0 = 0$, we have: $g(x) = f(0) + f'(0) \cdot x$.

a)

$f(x) = \sin(x)$
$g(x) = \sin 0 + \cos 0 \cdot x = x$

b)

$f(x) = e^x$
$g(x) = e^0 + e^0 \cdot x = 1 + x$

c)

$f(x) = \frac{x}{1+x^2}$
$g(x) = \frac{0}{1+0^2} + \frac{1+x_0^2 - x_0 \cdot 2x_0}{(1+x_0^2)^2} \cdot x = 0 + \frac{1}{1} \cdot x = x$

d)

$f(x) = e^x \cdot \sin x$
$g(x) = e^0 \cdot sin 0 + (e^{x_0} \cdot \cos x_0 + e^{x_0} \cdot \sin x_0) \cdot x = 0 + (e^0 \cdot \cos 0 + e^0 \cdot \sin 0)x = x$

Ex. 5

Birth rate: 4/day; $k_{birth} = 4$
Death rate: 1/day; $k_{death} = 1$
$\frac{dP}{dt} = (k_{birth} - k_{death}) \cdot P$
$\frac{dP}{dt} = (4 - 1)P$
$\frac{dP}{dt} = 3P$
$P(t) = P(0) \cdot e^{3t}$
We're given in the problem that $P(0) = 1000$. Therefore:
$P(t) = 1000 \cdot e^{3t}$