Math 19. Lecture 11
Nonlinear Stability Revisited

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1 Stability Criteria for a One-Component System

An equilibrium solution for

$$\frac{dx}{dt}(t) = f(x)$$

is a constant solution $x(t) = x_0$ such that

$$\frac{dx}{dt} < 0$$

where $x(t) > x_0$ and $x(t)$ is close to x_0 and

$$\frac{dx}{dt} < 0$$

where $x(t) < x_0$ and $x(t)$ is close to x_0. For example,

$$\frac{dx}{dt} = 9x - x^2 - 18 = (x-3)(6-x)$$

has equilibrium points at $x = 6$ and $x = 3$. The solution $x = 6$ is stable while the solutions $x = 3$ is unstable.

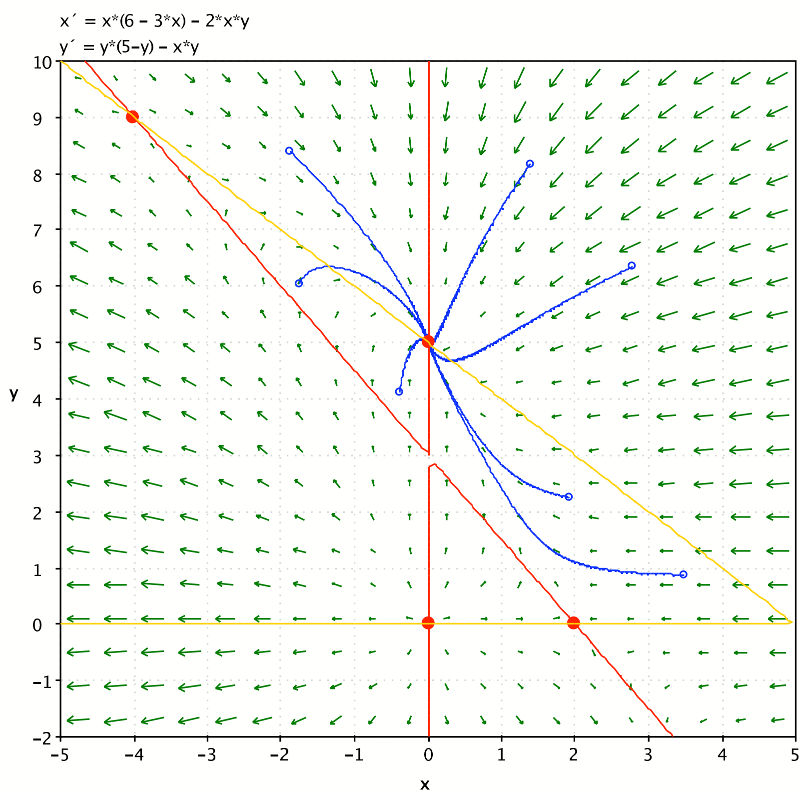
2 Stability Criteria for a Two-Component System

Recall the competition of species model. If two species compete for the same resources, then we can model this situation as

$$\begin{aligned} x' &= x(a_1 - b_1 x) - \alpha_1 xy \\ y' &= y(a_2 - b_2 y) - \alpha_2 xy. \end{aligned}$$

For example, consider the system

$$\begin{aligned} x' &= x(6 - 3x) - 2xy \\ y' &= y(5 - y) - xy. \end{aligned}$$



The x null clines are $x = 0$ and $y = -\frac{3}{2}x + 3$. The y null clines are $y = 0$ and $y = -x + 5$. The equilibrium points for this system are

$$\begin{pmatrix} -4 \\ 9 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix},$$

Since

$$D = \begin{pmatrix} 6-6x-2y & -2x \\ -y & 5-2y-x. \end{pmatrix}$$

Since

$$\begin{aligned} D(-4,9) &= \begin{pmatrix} 12 & 8 \\ -9 & -9 \end{pmatrix} \\ D(0,5) &= \begin{pmatrix} -4 & 0 \\ -5 & -5 \end{pmatrix} \\ D(0,0) &= \begin{pmatrix} 6 & 0 \\ 0 & 5 \end{pmatrix} \\ D(2,0) &= \begin{pmatrix} -6 & -4 \\ 0 & 3 \end{pmatrix}, \end{aligned}$$

$(0,5)$ is the only stable equilibrium point.

3 Multiple Integration

Recall single variable integration. For example,

$$\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}.$$

In two variables, let $R = [0,2] \times [0,1]$. Then

$$\iint_R x^2 y\,dA = \int_0^2 \int_0^1 x^2 y\,dydx = \int_0^1 \int_0^2 x^2 y\,dxdy$$

Homework

- Chapter 10. Exercises 1, 3, 4; p. 172.

Readings and References

- C. Taubes. *Modeling Differential Equations in Biology.* Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 10.

- “Hopes for the Future: Restoration Ecology and Conservation Biology,” pp. 153–166.