Math 19. Lecture 6
First-Order Systems

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1 Competing Species

Let x and y be the populations of two species at time t. We will assume that each species, in absence of the other, grows logistically:

$$\begin{aligned} x' &= x(a_1 - b_1 x) \\ y' &= y(a_2 - b_2 y), \end{aligned}$$

where a_1, a_2 are the growth rates of the two populations and a_1/b_1, a_2/b_2 are the carrying capacities. The simplest expression for reducing the growth rate of species x due to the presence of species y is to replace the growth factor $a_1 - b_1 x$ with $a_1 - b_1 x - \alpha_1$, where is a measure of the degree to which species y interferes with species x. The new system is now

$$\begin{aligned} x' &= x(a_1 - b_1 x) - \alpha_1 xy \\ y' &= y(a_2 - b_2 y) - \alpha_2 xy. \end{aligned}$$

2 Left and Right-Curling Snails

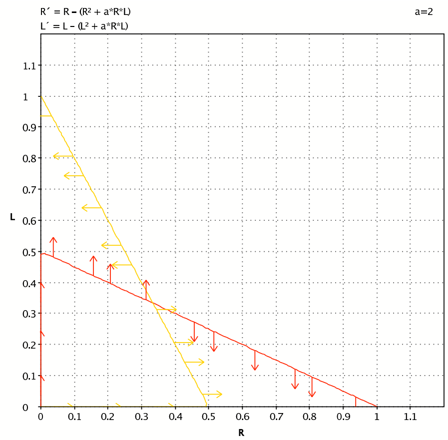
Let $L(t)$ denote the number (in millions) of left curling snails at time t and $R(t)$ the number of right-curling snails. The two populations compete for the same resources and might be governed by the following system of differential equations.

$$\begin{aligned} \frac{dR}{dt} &= R - (R^2 + aRL) \\ \frac{dL}{dt} &= L - (L^2 + aLR). \end{aligned}$$

The Case $a > 1$

Suppose $a = 2$. Then

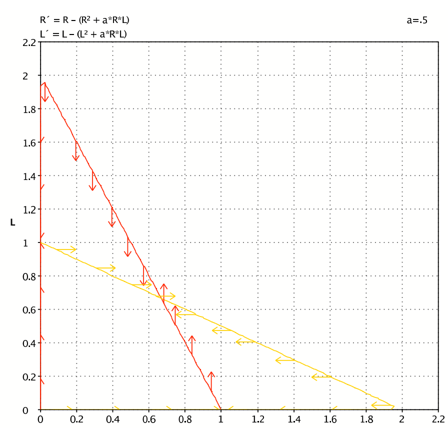
$$\frac{dR}{dt} = R - (R^2 + 2RL)$$
$$\frac{dL}{dt} = L - (L^2 + 2LR).$$



The Case $a < 1$

Suppose $a = 1/2$. Then

$$\frac{dR}{dt} = R - (R^2 + RL/2)$$
$$\frac{dL}{dt} = L - (L^2 + LR/2).$$



3 The Lotka-Volterra Equation

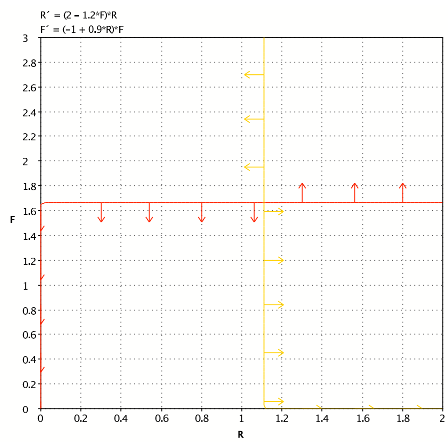
Suppose we have a population of rabbits, R, and foxes, F. The system

$$\begin{aligned}\frac{dR}{dt} &= (a - bR - cF)R \\ \frac{dF}{dt} &= (-d + eR)F.\end{aligned}$$

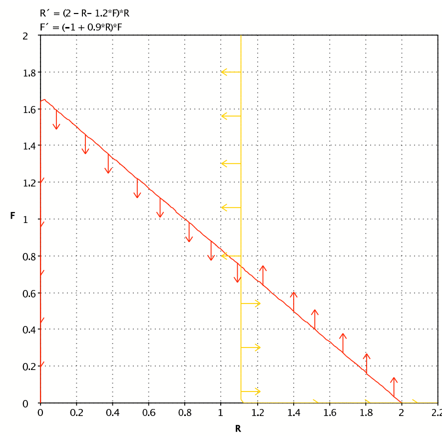
models the predator-prey relationship between the foxes and rabbits.

Consider the following systems.

- $$\begin{aligned}\frac{dR}{dt} &= (2 - 1.2F)R \\ \frac{dF}{dt} &= (-1 + 0.9R)F.\end{aligned}$$



- $$\begin{aligned}\frac{dR}{dt} &= (2 - R - 1.2F)R \\ \frac{dF}{dt} &= (-1 + 0.9R)F.\end{aligned}$$



Homework

- Chapter 5. Exercises 1; p. 86.

Readings and References

- C. Taubes. *Modeling Differential Equations in Biology.* Prentice Hall, Upper Saddle River, NJ, 2001. Chapter 5.
- "Left Snails and Right Minds," pp. 23–31.