# Math 19. Lecture 2
# A Calculus Toolkit

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## 1 The Coffee Problem

Two identical cups of dark liquid are left in a 70°F laboratory cool. At time $t = 0$, the first cup's temperature was 190°F, and was dropping at a rate of 12°F per minute. When did this cup's temperature fall to 130°F? The second cup was at 130°F after 10 minutes. Could this liquid be coffee?

## 2 Differential Equations

The most basic type of differential equation has the form

$$\frac{dy}{dx} = f(x, y).$$

In other words, if we know how a function changes, can we find the function?

A *solution* to a differential equation

$$\frac{dy}{dx} = f(x, y).$$

is a function $y = y(x)$ that satisfies the equation. For example, $y = x^4/4 + C$ is a solution to $y' = x^3$, where $C$ is an arbitrary constant. If we specify an initial condition, $y(0) = y_0$, then we can find a unique solution.

*In general, differential equations are very difficult to solve.* However, it is quite easy to check if a function is a actually a solution. For example,

$$y(t) = 70 + 120e^{-0.1t}$$

is a solution to the differential equation

$$\begin{aligned} \frac{dy}{dt} &= -\frac{1}{10}(y - 70), \\ y(0) &= 190. \end{aligned}$$

## 3 Newton's Law of Cooling

*An object cools at a rate proportional to the temperature difference between the object and its environment.* As a differential equation, Newton's Law of Cooling can be stated as

$$y' = k(y - T_e).$$

If we know the initial temperature at $t = 0$, then we have an *initial value problem*, which has a unique solution. We can state the initial condition as

$$y(0) = T_0.$$

## 4 Continuity and Differentiability in Biology

- If the true function under discussion jumps in value, then its replacement with a continuous function is reasonable when the experimental error is larger than any of the jumps.
- Once the step to a continuous function is made, the step to differentiability rarely adds trauma.

## 5 What Do You Need from Calculus?

- You need to understand the derivative and what it means.
- To be able to compute derivatives of elementary functions.
- To understand the definite integral and the relationship between antiderivatives and the definite integral.
- To be able to compute antiderivatives of elementary functions.
- To understand and be able to apply Taylor's Theorem.
- To understand how a curve can be parameterized.

## Homework

- None

## Readings and References


- C. Taubes. *Modeling Differential Equations in Biology.* Prentice Hall, Upper Saddle River, NJ, 2001, pp. 7, 81–85.
- A. Ostebee and P. Zorn. *Calculus from Graphical, Numerical, and Symbolic Points of View*, second edition. Harcourt, Orlando, FL, 2002.
- J. Stewart. *Calculus and Concepts*, second edition. Brooks/Cole, Belmont, CA, 2001.