Fractal analysis of SPM images

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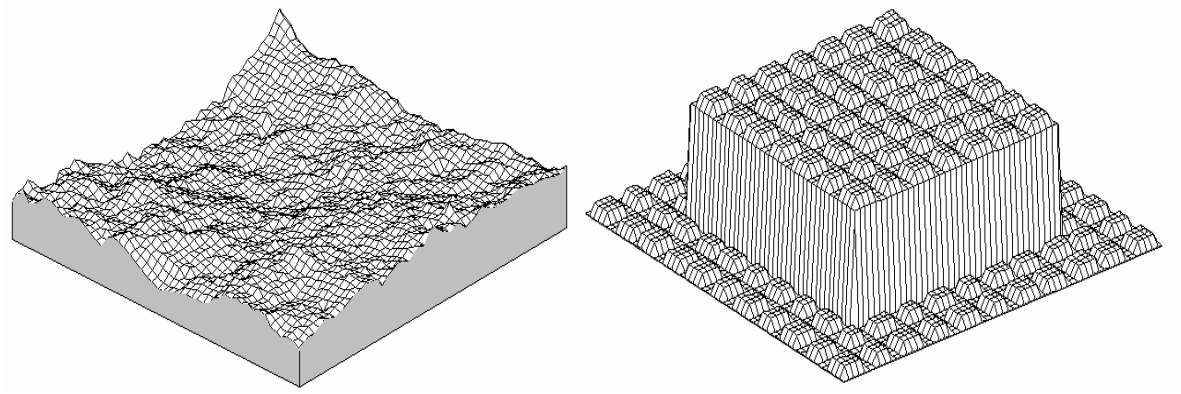
Fractal dimension (FD) is one of the possible ways for characterizing surfaces. There are many works showed the high usefulness and advantages of this parameter. Moreover FD has deep physical meaning. SPM makes the determining of fractal dimension especially easy. FD can be directly calculated from the surface geometry. But different methods of determining FD being applied to one surface very often give the different values and it is hard to say what is real FD of the surface. The main aim of this work is to critically evaluate the existing algorithms and to find out the possibility of applying them to various objects.

As the test objects we used the computer-simulated surfaces. The surfaces were simulated by two methods:

1) Menger's algorithm. This algorithm generates the *self-similar* fractal with known fractal dimension between 2 and approximately 2.6. (Figure 1, b). [1]

2) Random addition algorithm. This algorithm gives *self-affine* fractal surface with fractal dimension between 2 and 3. The surfaces were generated by means of this method look very naturally surfaces with fractal dimension between 2 and 3. (Figure 1, a) [2]



a) b)

Figure 1. Computer simulated surfaces with known value of fractal dimension.

We applied the following methods for calculating fractal dimension:

1) *Area-perimeter method with threshold at* $(Z_{max}+Z_{min})/2$ *level.* The intersection between a surface and the plane at the $(Z_{max}+Z_{min})/2$ level generates the self-similar lakes or islands. The scaling relation between perimeter (L) and area (A) of these lakes is:

$$L \sim A^{(D-1)/2}. \qquad (1)$$

Dimension D is calculated from the log-log plot of the perimeter versus area [2].

2) *Area-perimeter method with 100 thresholds from* Z_{min} *to* Z_{max}. The same as 1, but lakes are generated by 100 intersections at the levels between Z_{min} and Z_{max}.

3) *Plane box-counting method.* It is applied the usual box-counting method to the lakes boundaries generated in area-perimeter method 1.

4) *Variance method*. It is calculated the average variance as:

$$\sigma^2 = \left\langle \frac{1}{B^2 - 1} \sum_{i=1}^{B^2} (Z_i - \overline{Z})^2 \right\rangle, \qquad (2)$$

where B^2 is the number of data points in a box, Z_i – the height in each point, $\overline{Z}$ - average height in the box. Fractal dimension D is calculated form the relation between variance and boxsize B [3]:

$$\sigma^2(B) \sim B^{2(3-D)} \qquad (3)$$

5) *Nested box method.* This is the modification of box-counting method, where surface is covered by cubic boxes.

6) *Height-height correlation function method.* Fractal dimension is calculated from the roughness exponent α: $D=3-\alpha$, which is determined form height-height correlation function:

$$C(l) = \left[\left\langle \left(h(\boldsymbol{x}) - h(\boldsymbol{x'}) \right)^2 \right\rangle_x \right]^{\frac{1}{2}}, \ |x-x'|=l. \qquad (4)$$

Using the relation $C(l) \sim l^{\alpha}$ [4].

Table I. Average difference D_c - D_e between calculated D_c by the considered methods and estimated value D_e of surfaces.

Method	self- affine surface (figure 1, a)		self- similar surface (figure 1, b)	
	upper	lower	upper	lower
1	0,22	-0,08	-	-
2	0,13	-0,03	0,11	-0,55
3	0,13	-0,04	0,20	-0,50
4	0,18	-0.08	0,60	-0,03
5	0,05	-0,40	0,15	-0,40
6	0,38	-0,09	0,50	0,05

The results of calculations are in the Table I. They show, that all algorithms considered above give an significant error in the value of fractal dimension. Especially great error occurs on the surfaces with high fractal dimension. All methods tend to give higher value D_c in low-dimension surfaces and lower values on the surfaces with high fractal dimension. The most reliable methods for "real surfaces" as follows from results are lake-filling methods by levels, plane box-counting method and variance method.

[1] Кроновер Р.М. Фракталы и хаос в динамических системах. М.: Постмаркет, 2000.

[2] Федер Е. Фракталы. М.: Мир, 1991.

[3] Oden P.I., Majumdar A., Bhushan B., Padmanabhan A. Graham J.J. AFM Imaging, Roughness Analysis and Contact Mechanics of Magnetic-Tape and Head Surfaces, J. Tribol., **114** (1992), pp. 666-674

[4] Barabasi A.-L., Stanley H.E. Fractal concepts in surface growth. - Cambridge University Press, 1995.- 366 p.