THE ANALYSIS OF CT AND MR BRAIN IMAGES USING BOX COUNTING-TYPE METHODS

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Abstract: *Box Counting Dimension is an approximation of the theoretical Fractal Dimension. In this paper we use Box-Counting type methods in order to analyze CT and MR brain images. We first developed a computer application offering several tools, most of them based on Box Counting methods and variations for 2D and 3D structures. The conclusion is that the proposed algorithms can be used as medical tools to help the diagnosis process.*

Key words: *fractal dimension, box counting method, CT and MR brain images.*

INTRODUCTION

The use of fractal analysis is a known technique in clinical science. Particularly, the Fractal Dimension of various areas of tissues may help the process of diagnosis ([1],[2],[3],[4],[5]).

In this paper we propose an algorithm similar to Box Counting that provides helpful information to process CT and MR images and can be easily used as a base for developing an expert system.

1. FRACTAL DIMENSION AND BOX-COUNTING DIMENSION

The main ideas of fractal dimensions belong to Hausdorff. We give a brief introduction on what is usually known as Hausdorff dimension of a set embedded in the n-dimensional Euclidian space ([6])

Let $\Re^n = \{x = (x_1, x_2, ..., x_n), x_i\} \in \Re$ and let $d(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$ be the Euclidian distance. The diameter of a subset $U \subset \Re^n$ is defined by $diam(U) = \sup\{d(x,y) \mid x, y \in U\}$. Let $A \subset \Re^n$ and let $U_1, U_2, ...$ be an open cover of it. For every positive numbers s and ε we define

$$h_\varepsilon^s(A) = \inf\{\sum_i diam(U_i)^s\}$$

The s-dimensional Hausdorff measure of A is $h^s(A) = \lim_{\varepsilon \to 0} h_\varepsilon^s(A)$. It can be proved that there is a number $D_H(A)$ such that $h^s(A) = \infty$ if $s < D_H(A)$ and $h^s = 0$ if $s > D_H(A)$. The number $D_H(A)$ is the Hausdorff dimension of A and it can be zero, infinite or a a positive real number. We have also

$$D_H(A) = \inf\{s \mid h^s(A) = 0\} = \sup\{s \mid h^s(A) = \infty\}.$$

We give in the following some basic properties of the Hausdorff dimension:

(1) If $A \subset \Re^n$ then $D_H(A) \le n$.

(2) If $A \subset B$ then $D_H(A) \le D_H(B)$.

(3) If A is a countable set then $D_H(A) = 0$.

(4) If $D_H(A) < 1$ then A is totally disconnected.

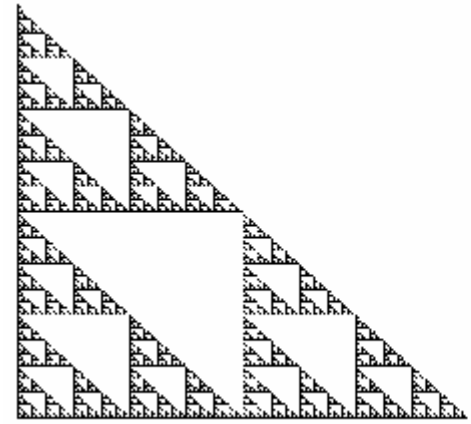
(5) The Hausdorff dimension of the Cantor set is log 2 / log 3, of the Sierpinski Gasket is log 3 / log 2 and of Sierpinski Carpet is log 8 / log 3.

There are major difficulties in evaluating the Hausdorff dimension in particular cases. For this reason there are simpler methods to approximate the Hausdorff dimension. One of the most efficient is the Box Counting method, ([6]).

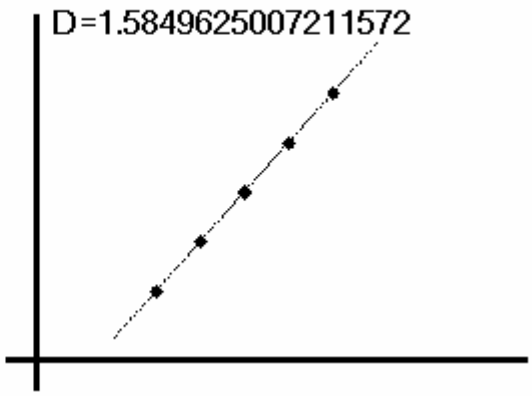
Let us consider a picture (structure). We cover the structure with a number of square boxes of size s. We count the number of the boxes which contain some part of the structure and let N(s) be this number. Clearly, if we increase the number of boxes or, equivalently, we decrease s to p, we obtain N(p). After this we make a diagram, on the Ox-axis we measure –log(s) and on Oy-axis we measure log(N(s)). In this way we obtain several points for different values of s. The Box Counting Dimension of the structures is defined as the slope of the regression line defined by the points on the diagram. The Box Counting Dimension is a good approximation of the Hausdorff Dimension (Fractal Dimension).

2. THE ALGORITHMS

We have implemented some box counting-type algorithms adapted to work with CT and MR images. First we have developed a classical Box Counting algorithm which can be applied on binary images of any size just for testing purposes. We have obtained very satisfactory results on some classical fractals.
As a sample we give below the results of our implementation for the Sierpinski Gasket and Sierpinski Carpet:

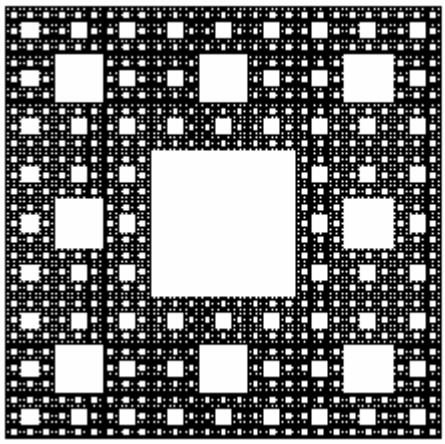


Sierpinski Gasket

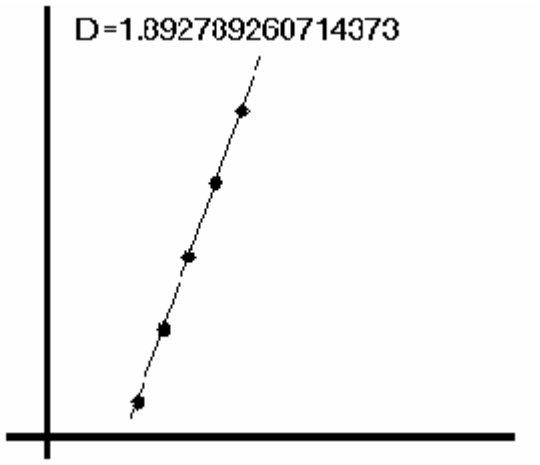


Regression Line

The fractal dimension of the Sierpinski Gasket is log 3 / log 2 ≈ 1.5850



Sierpinski Carpet

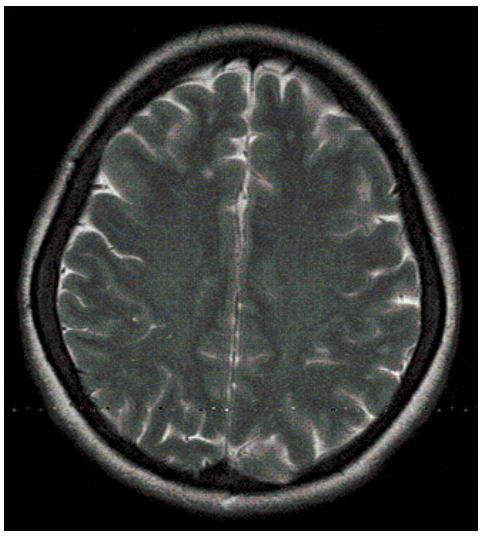


Regression Line

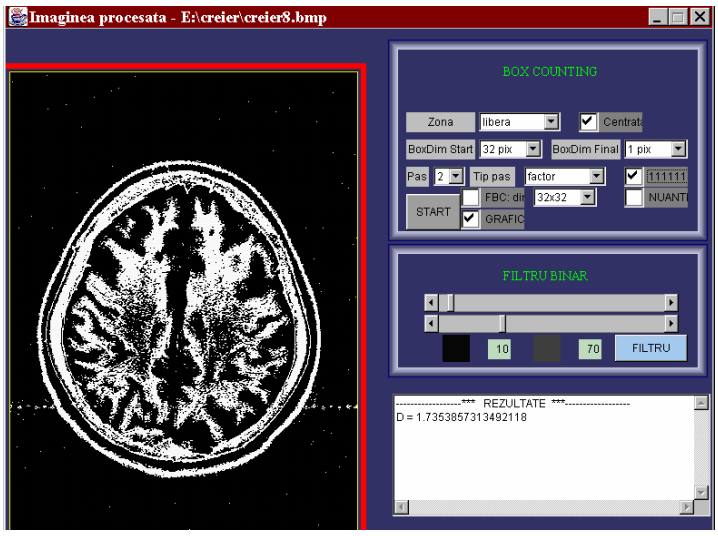
The fractal dimension of the Sierpinski Carpet is log 8 / log 3 ≈ 1.8928

Due to the fact that CT and MR images are both in gray-scale, the application of the previous Box Counting algorithm requires a binary filtering of the initial image into a black/white one. We also developed several filters, including color filters. Obviously, by applying different methods of filtering one obtain different results of the Box Counting Dimension for the same image.

We give below a sample of filtered brain image analyzed with the previous Box Counting algorithm.



MR Brain image



Filtered brain image. Fractal Dimension = 1.735…

In order to avoid the loose of information from initial image after filtering, we propose in this paper a Box Counting-like algorithm which can be applied on a gray-scale image. This algorithm is based on the fact that in the CT and MR images a higher density of the tissue is equivalent with lighter gray. Our idea was to associate to every pixel a weight proportional to its gray level. We resume the essential of the algorithm below.

Let us consider a brain image. We cover the image with square boxes as in the standard Box Counting algorithm. Let s_k be the size of the box used in covering at step k (therefore we have to compute $N(s_k)$ at this step). Let (x, y) be the coordinate of the upper-left corner of one of these boxes (let this be the box B_t^k). We now define m_t^k as the maximum of the weight values of the pixels contained in this box.

$$m_t^k = \max\{w_{i,j} \mid (i,j) \in ([x,y] \times [x+s_k-1, y+s_k-1]) \cap Z \times Z]\}$$

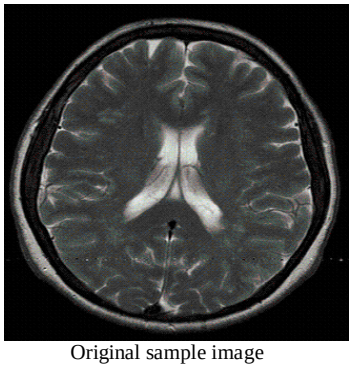
where $w_{i,j}$ is the weight associated to the pixel at coordinates (i, j).

Let $W_t^k = [m_t^k / s_k] + r_t^k$, where if $s_k \mid m_t^k$ then $r_t^k = 1$ else $r_t^k = 0$. Therefore $N(s_k) = \sum_t W_t^k$.

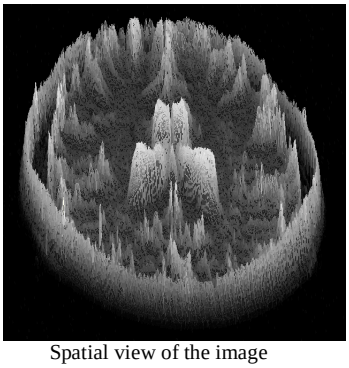
Next, the computation formula for D is the similar to the one in the classical algorithm. In this case, D will have a value in the [0,3] interval.

We shall refer to the number D as the Weighted Box Counting Dimension or WBCD.

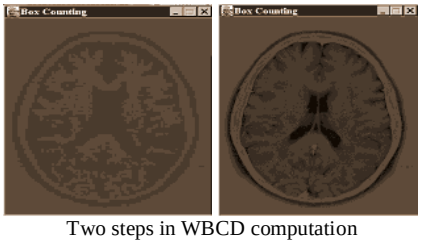
The following picture is a spatially representation of the volume to which essentially a 3D classical Box Counting algorithm is applied:



Original sample image



Spatial view of the image

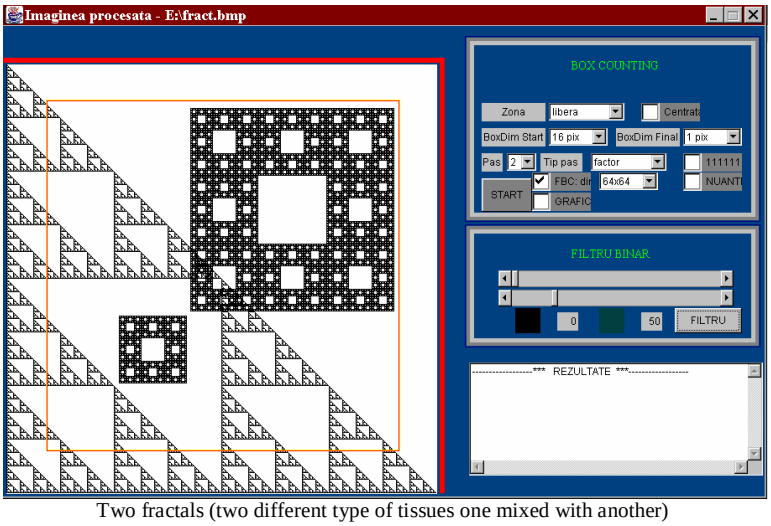


Two steps in WBCD computation

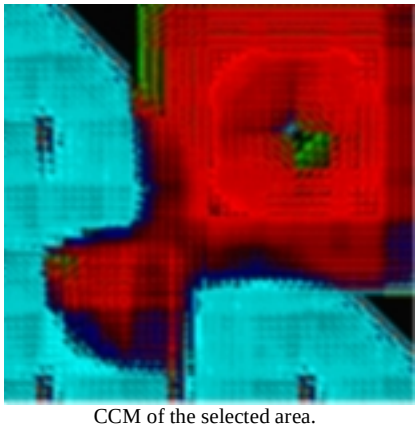
3. APPLICATIONS TO CT AND MR BRAIN IMAGES

Let us consider a brain image and let A be a pixel on this picture. Let K be a square centered at A. By using the previous algorithm we compute the WBCD of the square K and we associate a color to the pixel A according to this WBCD (the function which associates the color is a key part of the algorithm). In this way we obtain a map of level lines (we shall refer to this map as the Color Classification Map or CCM). This leads to a classification of different tissues according to the associated color. The use of the CCM in diagnosis requires a database with sufficient images. A new image is diagnosed by an expert system that compares the Color Classification Maps.

To show the idea behind Color Classification Map, we shall apply the described algorithm on pictures containing some known fractals. Different structures must have different colors.

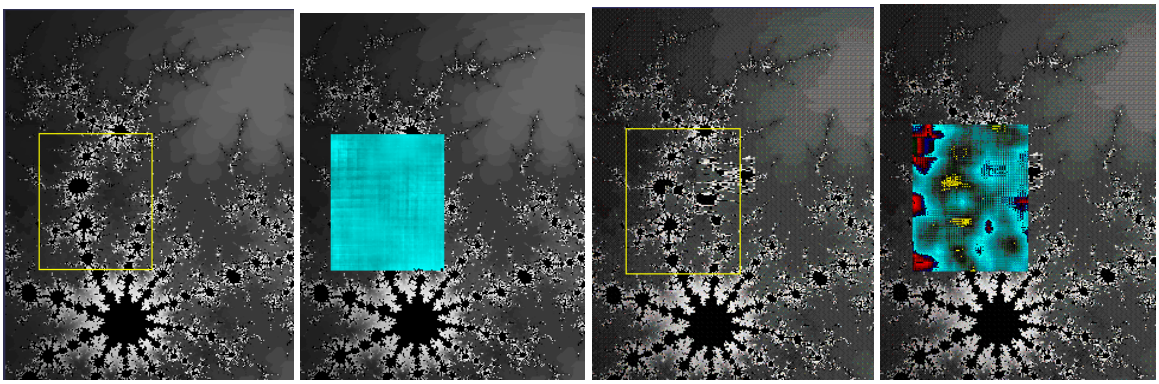


Two fractals (two different type of tissues one mixed with another)

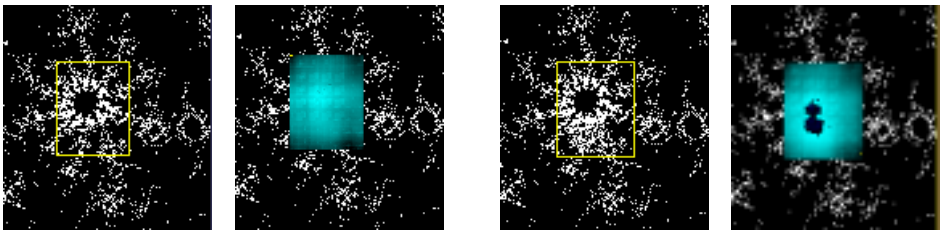


CCM of the selected area.
Discontinuities and "parasite zones" are caused by poor original image resolution

We notice that CCM reveals two types of fractals (tissues) in the analyzed image.
Next, we make the CCM of a random area from a Julia Fractal:

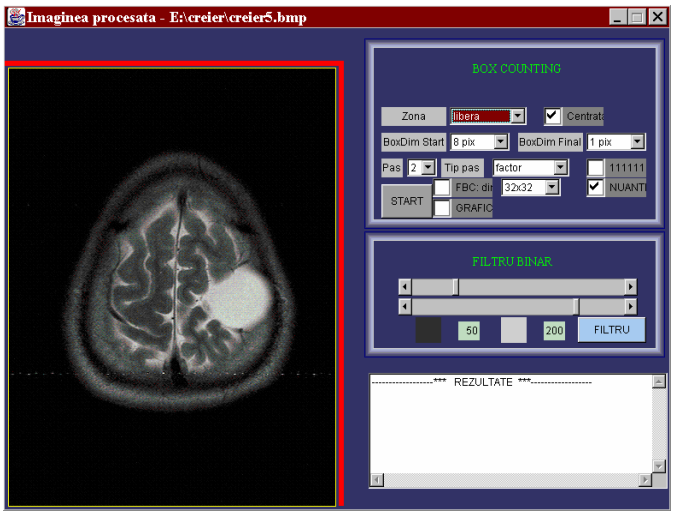


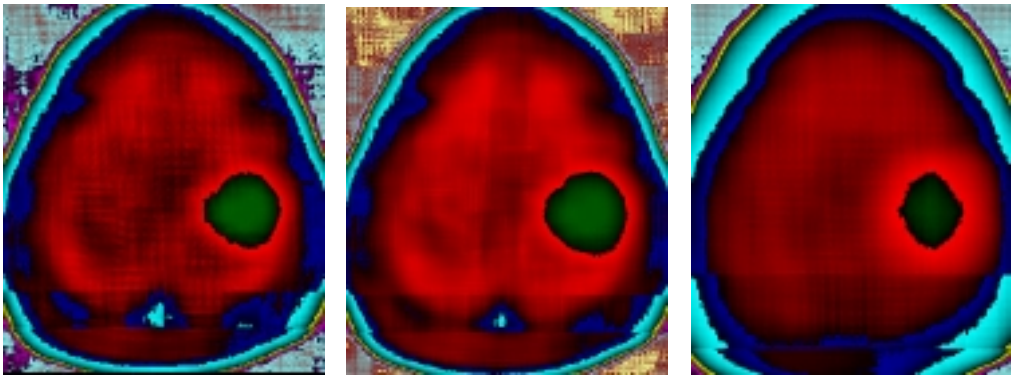
In the above sample the left picture is a Julia set. The second picture shows the CCM of the selected area. In the third picture we made in the original structure some "anomalies" (scaled parts from itself have been added). The associated CCM is strongly non-homogenous, revealing the perturbation. The most important fact is that on the "real" image the perturbation is not evident, while on the associated CCM the perturbation becomes obvious.



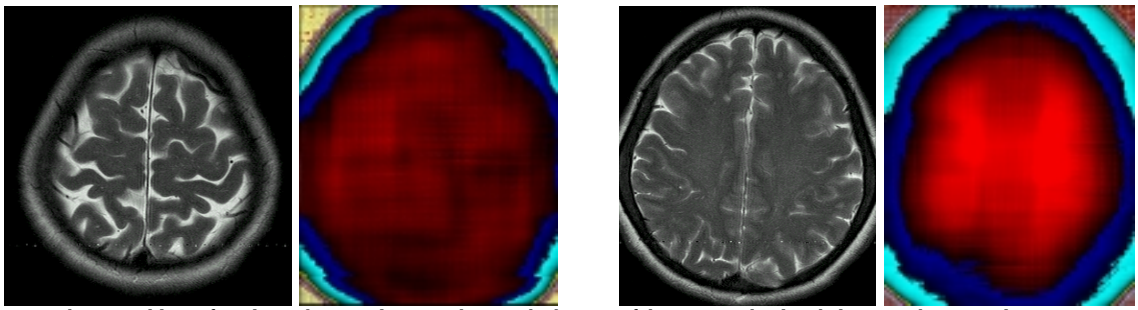
This is another example of the same type as above, but the perturbation is of a “parasite” type, i.e. the new structure belongs to another fractal**.** (We simply used a spray tool to insert some extra white pixels in the image).

In the following we present several brain images and their associated CCM’s. One can notice that the CCM reveals the “modified” structures (tumors), if any.

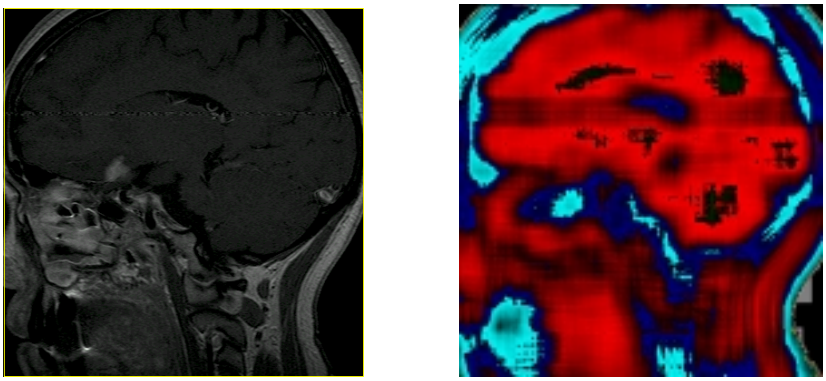




CCM’s at different parameters (this parameters can be use in analysis too)



Note: There is a horizontal line of random white pixels somewhere at the bottom of the image. That line belongs to the original MR image (probably a capture error) and produces an anomaly in the CCM around it also.



We notice different tissues with different WBCD on the CCM

CONCLUSIONS

The method of the Color Classification Map is a useful tool that can help in the process of the diagnosis. During the study the authors observed that some significant details of CT or MR images which were “invisible” on the initial image became obvious on the CCM’s. More precisely, interesting modifications (pathological or not) which were not detected on the real image, became clearly distinct zones on CCM simply because of the their Fractal Dimension (WBCD) was different. This simple fact requests a further medical investigation of that zone which could improve the diagnosis.

We are currently developing an expert system designed to be an assistant for the diagnosis process. It will be also capable to provide a diagnosis based on MR or CT images, the associated CCM’s and some CCM’s information synthesis algorithms.

The authors observed some interesting relations between the aspect of the boundaries of the areas in the CCM and the evolution of the associated tissues (in particular, tumors). This will be the objective of a further application

REFERENCES

[1] C. Balestra et al./ HarFA - *Harmonic and Fractal Image Analysis*, pp. 25 – 29, 2002

[2] T.N. Behar, Analysis of fractal dimension of O2A glial cells differentiating in vitro, *Methods* 24, 331-339, 2001

[3] C.B. Caldwell, E.L. Moran, E.R Bogoch, Fractal dimension as measure of altered trabecular bone in experimental inflammatory arthritis, *J Bone miner Res* 13, 978-985, 1998

[4] S.S. Cross, Fractals in Pathology. *Journal of Pathology* 182, 1-8, 1997

[5] P. Luzi, G. Bianciardi, C. Miracco, M.M. De Santi, M. Del Vecchio, L.Alia, P.Tosi, Fractal analysis in human pathology. *Ann N Y Acad Sci* 879, 255-257, 1999

[6] H-O. Peitgen, H. Jurgens, D. Saupe, Chaos and Fractals – New Frontiers of Science, Springer, 2000